

- (c) A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw ? 7,7,6

6. (a) Solve the equation :

$$\begin{cases} x_n = 3x_{n-1} - 4n, n \geq 1 \\ x_0 = 2. \end{cases}$$

(b) Find the ordinary generating function for the sequence $\{a_n\}$ $n \geq 0$ satisfying

$$a_n = 2a_{n-1} + 1, \quad n \geq 1, \quad a_0 = 0 \quad 10,10$$

SECTION—D

7. Define switching functions and Logic circuits in the context of Boolean algebra. Discuss various applications of Boolean algebra in logic circuits and switching functions, by taking examples. 20
8. (a) What are integral domains and fields ? Can you think of a ring that isn't an integral domain or an integral domain that isn't a field ?
- (b) What is the characteristic of a ring ? What are sub-rings and ideals ? What's the difference ? 10,10

Exam. Code : 206701

Subject Code : 4785

M.Sc. Computer Science 1st Semester (Batch 2021-23)

DISCRETE STRUCTURES

Paper—MCS-104

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt FIVE questions in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Define $g : \mathbb{Z} \rightarrow \mathbb{Z}$ by $g(n) = 3n - 1$, where \mathbb{Z} is the set of integers.
- (i) Is g one-to-one ?
- (ii) Is g onto ?
- (iii) Suppose that $g : \mathbb{R} \rightarrow \mathbb{R}$ and $g(x) = 3x - 1$ for all real numbers x . Is g onto ?
- (b) There is a 6-digit Personal Identification Number (PIN) encoded in each bank card for security reasons. Find the number of possible PINs :
- (i) with repeated digits allowed,
- (ii) with no repeated digits. 10,10

2. Prove each of the following identities from the basic algebraic rules for sets :

(a) If A, B and C are subsets of U, then

$$(A - B) - C = A - (B \cup C).$$

(b) If A, B and C are subsets of U, then

$$(A - B) - C = (A - C) - B.$$

(c) If A and B are subsets of U, then

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B).$$

7,7,6

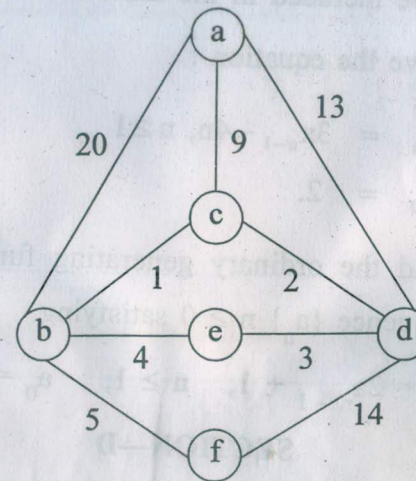
SECTION—B

3. (a) What is an undirected graph ? Prove that an undirected graph has even number vertices of odd degree.

(b) Define a binary relation D from $\underline{10} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ to $\underline{10}$ as follows : For all x, y in $\underline{10}$, $x D y$ if $x < y$ and x divides y. How many edges are there in the directed graph of this relation ? Explain.

10,10

4. (a) Find minimum spanning tree for the following undirected weighted graph starting from vertex 'a' :



(b) Give an example of a graph which is :

(i) Eulerian but not Hamiltonian

(ii) Hamiltonian but not Eulerian. 10,10

SECTION—C

5. (a) How many numbers are there between 99 and 1000 having at least one of their digits 7 ?

(b) If all permutations of the letters of the word AGAIN are arranged in the order as in a dictionary. What is the 49th word ?